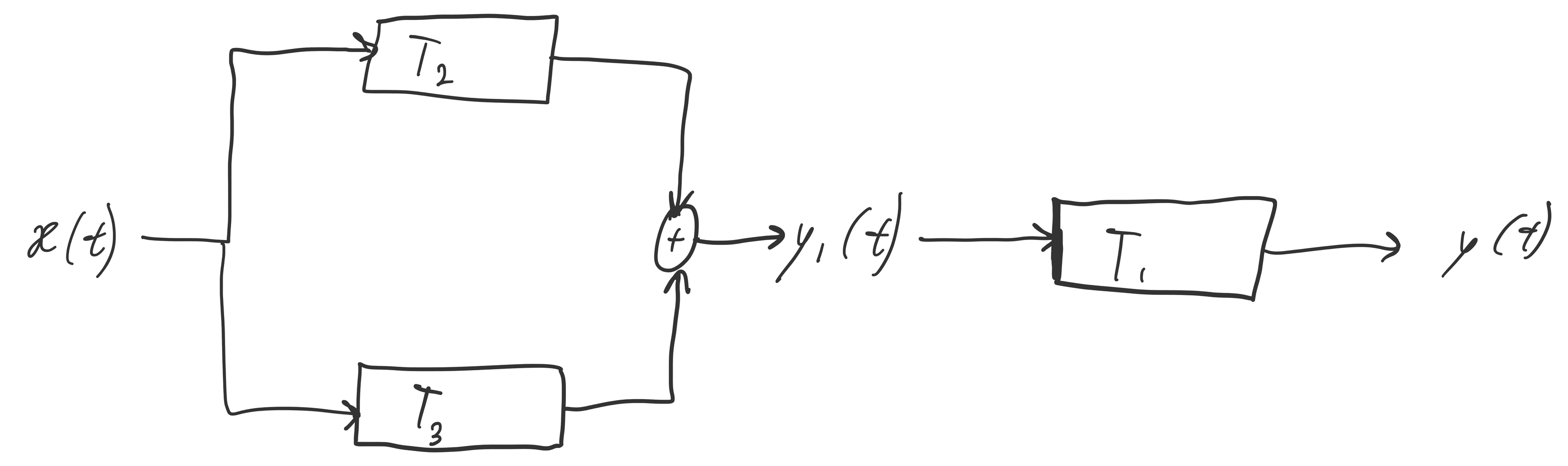
Exercises Chapter 10

Tuesday, 14 January 2020 11:15

10.1



10.2

$$T[\alpha x_1+\beta x_2] = \frac{d}{dt}(\alpha x_1+\beta x_2) = \frac{d}{dt}(\alpha x_1) + \frac{d}{dt}(\beta x_2) = \alpha\frac{d}{dt}x_1 + \beta\frac{d}{dt}x_2$$
$$= \alpha T[x_1] + \beta T[x_2]$$

Hence, the system is linear

$$T[\alpha x_1+\beta x_2] = \int_0^t \alpha x_1(\tau)+\beta x_2(\tau)\,d\tau$$
$$= \alpha\int_0^t x_1(\tau)\,d\tau + \beta\int_0^t x_2(\tau)\,d\tau$$
$$= \alpha T[x_1] + \beta T[x_2]$$

Hence, the system is linear

$T[x] = x(t) + c$ with $c \neq 0$ is not linear, as

$$T[\alpha x_1+\beta x_2] = \alpha x_1+\beta x_2 + c \neq \alpha x_1+\beta x_2+(\alpha+\beta)c = \alpha(x_1+c)+\beta(x_2+c) = \alpha T[x_1]+\beta T[x_2]$$

whenever $\alpha \neq -\beta$

$$T[\alpha x_1+\beta x_2] = (\alpha x_1+\beta x_2)f(t) = \alpha x_1 f(t)+\beta x_2 f(t) = \alpha T[x_1]+\beta T[x_2]$$

Hence, this system is linear

in general, write $x_1(t)$ and $x_2(t)$

10.3

~~$y(t) = D[x(t)]$~~

~~$y(t-\tau) = \frac{d}{dt}x(t-\tau)$~~

$$D S_\tau[x] = \frac{d}{dt}x(t-\tau)\Big|_t = S_\tau D[x] = \frac{d}{dt}x(t)\Big|_{t-\tau}$$

because D and S_τ commute, D is time-invariant

10.4

1 no, see theory just above: $x(t) = \pm y(t)$

2 yes, $y(t) = \alpha x^3(t) + \beta$

$$\alpha x^3(t) = y(t) - \beta$$
$$x^3(t) = \frac{1}{\alpha}(y(t)-\beta)$$
$$x(t) = \sqrt[3]{\frac{y(t)-\beta}{\alpha}}$$

3 no, the heaviside function (and hence, a product with it) is not injective, hence not bijective and hence not invertible

A system is BIBO-stable if and only if, given an arbitrary bound for its input, it is possible to find a bound *(maximum absolute value)* for its output (possibly depending on the bound for its input) which does not depend on any variable (especially *t*).

?

$T[x(t)]$

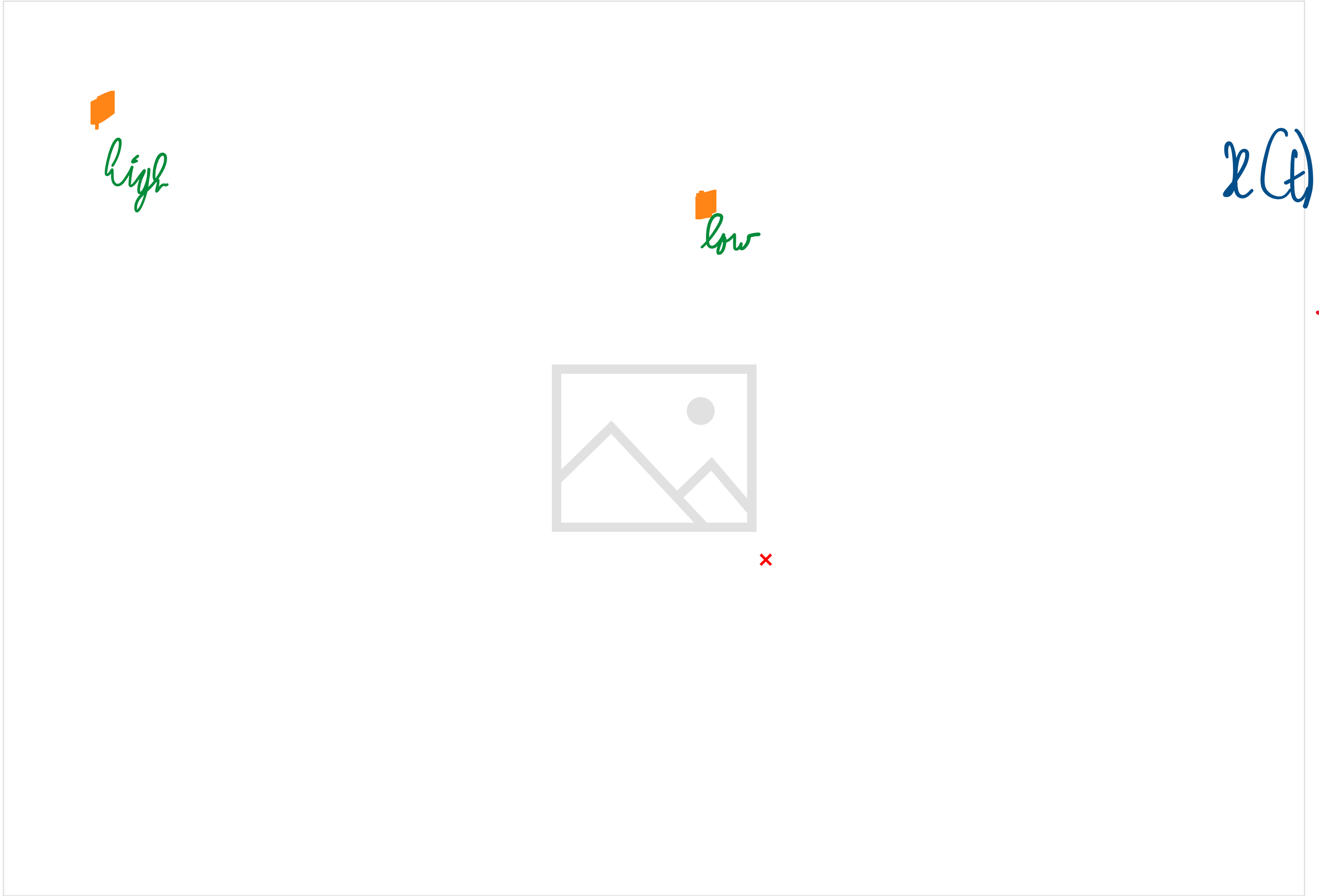
$x(t)$ $y(x(t))$

y

memory examples?

common mistakes? (vectors?)

3.1? exam (2017 - 4)



$x(t) = x_1, x_2, x_3$

$x\left(\sum_i x^i(t)\right)$